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Mathematical Model and Analysis of the Effects of Social Determinants of Health Contributing to Teenage fertility in Kenya.

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ABSTRACT

Teenage pregnancy remains a significant public health challenge in Kenya, with adolescent sexual and reproductive health needs largely unmet, especially among marginalized and vulnerable groups. Although the 2022 Kenya Demographic Health Survey (KDHS) reports a slight decline in teen pregnancy from 18% in 2014 to 15% in 2022, progress toward achieving Kenya Vision 2030 remains slow. Understanding the effects of social determinants on teenage fertility through rigorous mathematical modeling is essential for informing effective interventions and policy actions. This study develops and analyzes a deterministic differential equations model incorporating demographic, socio-economic, and socio-cultural factors affecting adolescent fertility among females aged 15-19 years using KDHS 2022 data. In this model, qualitative theory of differential equations and stability analysis has been used to assess the impact of proximate determinants on fertility outcomes. Stability analysis reveals that both eigenvalues, $\lambda 1 = -\beta$ and $\lambda 2 = -\delta$, are negative, establishing that the system's equilibrium is asymptotically stable. Sensitivity analysis further identifies that the decay parameters β and δ exert strong negative influence on the fertility reproduction potential R0, highlighting critical social determinants that significantly influence teenage fertility rates. These insights allow prioritization of intervention strategies targeting the most impactful factors. Importantly, the model demonstrates that targeted policy interventions can effectively shift the system to a new, lower equilibrium teenage fertility rate, providing valuable guidance for policymakers. Ethical approval for this study was obtained from the National Commission for Science, Technology and Innovation in Kenya, ensuring adherence to national research ethics guidelines. The findings provide robust, evidence-based insights for government and stakeholders to design impactful strategies to reduce teenage pregnancies in Kenya.

Keywords: Teenage Pregnancy, Adolescent Fertility, Mathematical modeling, Risk factors, Sensitivity Analysis, Stability analysis

1. INTRODUCTION

Adolescent fertility, defined as childbearing and pregnancy prior to age 20, constitutes a significant imped- iment to global development,particularly in low and middle -income countries (WHO,2021). Worldwide, adolescents suffer from a disproportionate share of reproductive health problem. The World Health Or- ganization (WHO) estimates that 16 million adolescent girls give birth each year, 95% of these births occurring in less developed nations, underscoring the urgency of addressing this issue (Alemayehu et al, 2010; WHO, 2022). Teenage pregnancy not only poses substantial health risks to both mother and child but also impedes educational attainment and perpetuates cycles of poverty Reducing adolescent fertility and addressing the multiple factors underlying it are essential for improving sexual and reproduc- tive health and, ultimately, the social and economic well being of adolescent girls (UN,2013; UN,2019). Achieving the Sustainable Development Goals (SDGs) necessitates a concerted effort to mitigate adoles- cent fertility rates,especially in regions lagging behind. Sub-Saharan Africa bears the highest burden of adolescent fertility,with rates significantly exceeding global average at 101

births per 1,000 women (West- off & Cross, 2006). In Kenya, the teenage birth rate declined from 168 live births per 1,000 in 1977/78 to 96 in 2014, yet teen pregnancy remains prevalent at 18% [KNBS & ICF Macro, 2014; KNBS & ICF Macro,

2022)]. Various factors, including level of education, use of contraceptives, wealth index, marital status, and cultural aspects, are accountable for adolescent fertility, with disparities existing within and across countries (Eyasu, 2016; Raharjo et al, 2019; Olorinola, 2016; Nyarko, 2012).

Moreover, teenage pregnancy has series long term problems that affect the girls themselves and their community. It leads them to less educational attainment , disease exposure and economic problems, psychosocial and eco- nomic dangers , including thwarting their reproductive health, child birth, career growth, keeping them in vicious cycle of poverty (many come from already poor family), and overall limiting of their capa- bilities, opportunities and choices. This will not end by themselves rather their siblings are also prone to these problems and are more likely to give birth as teenagers, face unemployment and engage in criminal acts at some time during their adolescence (Oluwatayo, 2022; Oigo, 2019; Tigabu et al, 2021). Bongaart's proximate determinants framework provides a valuable lens for analyzing the roles of contraceptive use, induced abortion, breastfeeding and marriage prevalence (Bongaarts & Potter, 2013). In Kenya, a plateau in contraceptive use among adolescents has driven rising fertility rates, with the proportion of youth objecting to family planning increasing from 13.4% to 22.4% between 1998 and 2003 (Westoff & Cross , 2006). Additionally, rural adolescents, the less educated, and those from less afflu- ent families are more prone to experience early childbearing, more often unwanted (Obare et al, 2016). Beyond individual consequences, adolescent pregnancy carries broader social and economic implications, including stigma, school dropouts, and reduced future incomes, with child marriage reducing lifetime earnings by 9% and negatively impacting national economies (Sinisa, 2018). However,a significant gap remains in understanding the dynamic interactions between these determinants and the broader social context in which adolescents make reproductive health decisions. Moreover, few studies have employed mathematical modeling to quantitatively assess the relative importance of various social determinants and to predict the impact of policy interventions on teenage fertility rates in the Kenyan context. This study addresses this gap by developing a mathematical model to analyze the effects of social determinants on teenage fertility in Kenya. Unlike purely statistical approaches, our model allows for the simulation of dynamic interactions between key factors and the assessment of long term impacts. By incorporating the proximate determinants framework and drawing upon data from the 2022 Kenya Demographic Health Survey (KDHS), we aim to identify the most influential drivers of adolescent fertility and to evaluate the potential effectiveness of different policy scenarios. The findings for this study provides valuable insights for policymakers in designing targeted interventions to reduce teenage pregnancies and improve adoles- cent sexual and reproductive health outcomes, contributing to the achievement of the SDGs and Vision 2030 in Kenya. Furthermore, the Mathematical framework developed in this study can be adapted and applied to other developing countries facing similar challenges, offering a generalizable tool for under- standing and addressing adolescent fertility. The use of stability analysis also offers insights into the long term behavior of the system, identifying the conditions that lead to sustainable reductions in teenage pregnancy rates. The combination of empirical data with a dynamic mathematical model represents a unique contribution to the field and offers a powerful tool for evidence based policy making.

Methodology

Methods of Solution

The model for Teenage fertility was developed and analyzed for stability using the Jacobian matrix method. The system of differential equations was linearized around the equilibrium point to examine local behavior. By evaluating the eigenvalues of the Jacobian matrix, it was established that eigenval- ues were negative showing that the model was locally asymptotically stable. This approach provides a framework for understanding how small perturbations in the system variables influence long-term fertil- ity outcomes, thereby guiding the assessment of intervention impacts on adolescent fertility dynamics. In addition to stability analysis using the Jacobian matrix, the analytical solutions of the model for the key variables are derived to explicitly describe their temporal behavior and convergence towards equi- librium. This analytical framework facilitates understanding of the system dynamics over time under various initial conditions. Furthermore, sensitivity analysis is conducted to quantify the influence of model parameters on the fertility reproduction potential R_0, which identifies key socio-demographic and biological determinants with the greatest influence. These combined approaches provide a comprehensive methodology for assessing both the stability and responsiveness of the teenage fertility system to changes in underlying determinants.

Model Assumptions

The following assumptions are adopted in the formulation and analysis of the compartmental fertility model involving demographic (D), socio-economic (S), socio-cultural (C), proximate determinants (P), and fertility outcome (N) compartments:

Structural Assumptions

The flow of influence is unidirectional from D, S, and C to P, and from P to N, with no feedback loops.

Individuals are not transferred between compartments; rather, variables influence other vari- ables as inputs.

No feedback from the fertility outcome N to the upstream compartments.

Variable Assumptions

Demographic factors (D) such as age at first sex and marital status are assumed fixed over the modeling period.

Socio-economic variables (S) including education level, employment, and residence type are considered stable.

Socio-cultural characteristics (C) such as religion and ethnicity are treated as constant back- ground traits.

Proximate determinants (P) are modeled as a composite of three non-interacting sub-factors: contraceptive use (P_1), proportion married (P_2), and postpartum infecundability (P_3).

Fertility outcome (N) accumulates over time based on the contributions from proximate de- terminants and is non-decreasing.

Parameter Assumptions

All influence parameters (α_1, α_2, α_3) and fertility weights (γ_1, γ_2, γ_3) are constant over time.

The decay parameters β (for P) and δ (for N) capture natural regulatory or behavioral constraints and are assumed fixed.

The effect of each background factor on P is linear and additive, with no cross-effects or multiplicative interactions.

Population Assumption

i. The population is assumed closed, with no migration, mortality, or explicit age progression.
ii. Fertility dynamics are only affected by internal socio-demographic and cultural characteristics.

Data and Measurement Assumptions

i. All model compartments are either directly measurable or estimable from available demo-graphic and health survey data.
ii. Measurement error, reporting bias, and missing data are assumed negligible.

Survey indicators are assumed to adequately capture the conceptual dimensions of each compartment

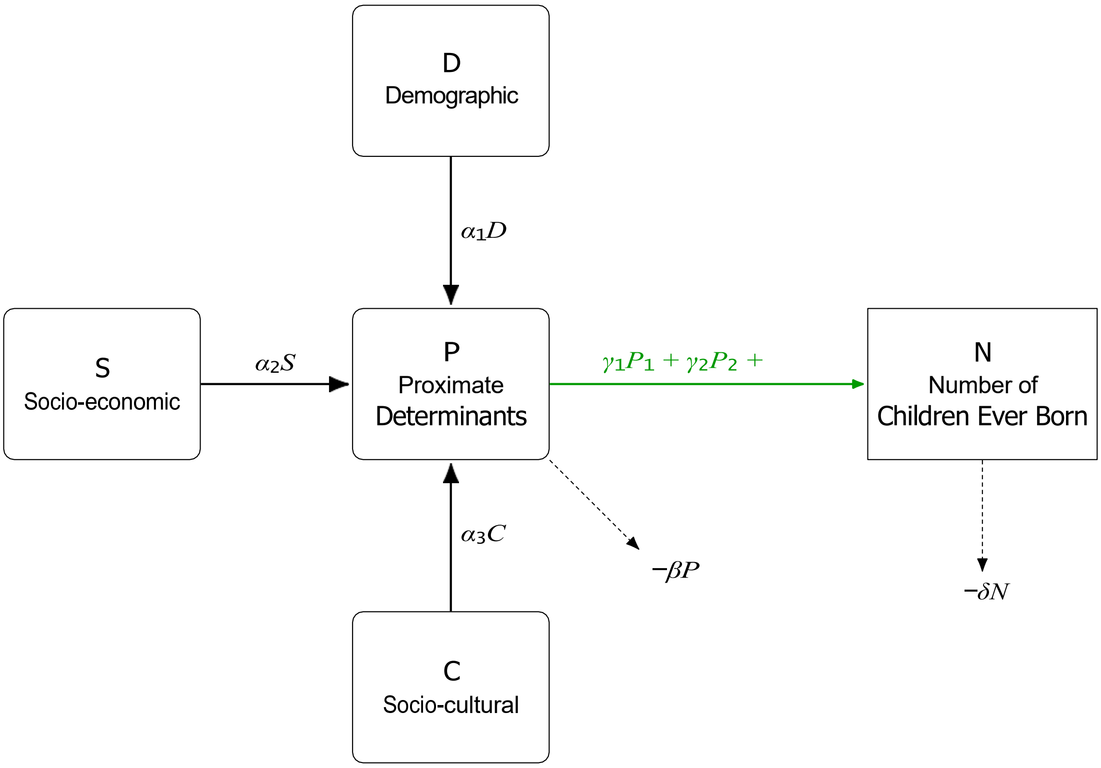

Figure 1: Compartmental diagram showing influence of demographic, socio-economic, and socio-cultural factors on proximate determinants and fertility outcome.

1.1 Model Equations

We can construct a system of equations representing the relationships between different factors and the dependent variable (Number of Children Ever Born)and define the variables as follows:

Let: D_1, D_2, D_3 be Demographic Factors(e.g., Age at first sex, Current age, Current marital status). S_1, S_2, S_3, S_4 be Socio-Economic Factors (e.g., Education level, Residence type, Wealth index, Employ- ment status, Use of contraceptives).

C_1, C_2, C_3 be Socio-Cultural Factors (e.g., Religion, Radio frequency, Ethnicity).

P_1, P_2, P_3 be Proximate Determinants (e.g., Contraceptive use, Proportion married, Postpartum infe- cundability).

N be the Dependent Variable (Number of children ever born).

1.2 Differential Equations in Fertility Dynamics

We define the Variables.

Let: $D(t)$ represent demographic factors (age at first sex, current age, marital status).

$S(t)$ represent socio-economic factors (education, residence, wealth, employment, contraception).

$C(t)$ represent socio-cultural factors (religion, radio, ethnicity).

$P(t)$ represent proximate determinants (contraceptive use, proportion married, postpartum infecund- ability).

$N(t)$ represent the dependent variable (number of children ever born).

Since proximate determinants mediate the effect of background factors, we introduce functions f and g:

$$\frac{dP}{dt} = f(D, S, C),$$

$$\frac{dN}{dt} = g(P) \quad (1)$$

1.3 Differential Equations

1.3.1 Proximate Determinants Dynamics

The rate of change of proximate determinants depends on background factors:

$$\frac{dP}{dt} = \alpha_1 D + \alpha_2 S + \alpha_3 C - \beta P \quad (2)$$

where:

i. $\alpha_1, \alpha_2, \alpha_3$ are weights representing the influence of each background factor on proximate determinants.

ii. β represents the decay (e.g., societal changes reducing influence over time).

1.3.2 Fertility Rate Dynamics

The rate of change of children ever born depends on proximate determinants:

$$\frac{dN}{dt} = \gamma_1 P_1 + \gamma_2 P_2 + \gamma_3 P_3 - \delta N \quad (3)$$

where:

i. P_1, P_2, P_3 correspond to contraceptive use, proportion married, postpartum infecundability.

ii. $\gamma_1, \gamma_2, \gamma_3$ are fertility impact factors.

iii. δ accounts for delays or external factors reducing fertility rates.

The system of differential equations is:

$$\begin{pmatrix} \frac{dP}{dt} \\ \frac{dN}{dt} \end{pmatrix} = \begin{pmatrix} \alpha_1 D + \alpha_2 S + \alpha_3 C - \beta P \\ \gamma_1 P_1 + \gamma_2 P_2 + \gamma_3 P_3 - \delta N \end{pmatrix} \quad (4)$$

1.4 Positivity Analysis

From the analytical solution:

$$P(t) = \frac{\alpha_1 D + \alpha_2 S + \alpha_3 C}{\beta} + \left(P_0 - \frac{\alpha_1 D + \alpha_2 S + \alpha_3 C}{\beta}\right) e^{-\beta t} \quad (5)$$

As $t \to \infty$:

$$P(t) \to P^* = \frac{\alpha_1 D + \alpha_2 S + \alpha_3 C}{\beta} \quad (6)$$

Similarly:

$$N(t) = \frac{\gamma_1 P_1 + \gamma_2 P_2 + \gamma_3 P_3}{\delta} + \left(N_0 - \frac{\gamma_1 P_1 + \gamma_2 P_2 + \gamma_3 P_3}{\delta}\right) e^{-\delta t} \quad (7)$$

As $t \to \infty$:

$$N(t) \to N^* = \frac{\gamma_1 P_1^* + \gamma_2 P_2^* + \gamma_3 P_3^*}{\delta} \quad (8)$$

Conclusion: Both variables are bounded by their equilibrium values plus initial deviations.

1.4.1 Finding Equilibrium Values for the System

To find equilibrium values, we set the derivatives to zero (steady-state conditions):

$$\frac{dP}{dt} = 0, \quad \frac{dN}{dt} = 0 \quad (9)$$

Step 1: Equilibrium of Proximate Determinants

$$0 = \alpha_1 D + \alpha_2 S + \alpha_3 C - \beta P \quad (10)$$

Solving for P^*:

$$P^* = \frac{\alpha_1 D + \alpha_2 S + \alpha_3 C}{\beta} \quad (11)$$

Step 2: Equilibrium Number of Children Born

$$0 = \gamma_1 P_1 + \gamma_2 P_2 + \gamma_3 P_3 - \delta N \quad (12)$$

Since P is known, substitute P_1, P_2, P_3 as components of P^*:

$$N^* = \frac{\gamma_1 P_1^* + \gamma_2 P_2^* + \gamma_3 P_3^*}{\delta} \quad (13)$$

Interpretation of Equilibrium

i. The equilibrium values P and N depend on background factors D, S, C.

ii. If background factors increase, proximate determinants change, leading to a shift in fertility rates.

iii. If contraceptive use increases, N^* will decrease, stabilizing fertility at a lower level.

iv. If all external influences are constant over time, the system remains at equilibrium.

1.5 Basic Reproduction Number (R_0)

Definition 2.1 (Fertility Reproduction Potential). *In the context of fertility dynamics,* R_0 *represents the fertility reproduction potential - the expected number of children born per unit of proximate determinant influence.*

$$R_0 = \frac{\text{Rate of fertility increase}}{\text{Rate of fertility decrease}} = \frac{(\gamma_1 + \gamma_2 + \gamma_3) \cdot P*}{\delta} \tag{14}$$

Substituting $P*$:

$$R_0 = \frac{(\gamma_1 + \gamma_2 + \gamma_3)(\alpha_1 D + \alpha_2 S + \alpha_3 C)}{\beta\delta} \tag{15}$$

Interpretation:

i. $R_0 > 1$: High fertility regime (sustainable population growth)

ii. $R_0 < 1$: Low fertility regime (population decline tendency)

iii. $R_0 = 1$: Replacement level fertility

1.6 Low Fertility Equilibrium

Definition 2.2 (Low Fertility Equilibrium). *The low fertility equilibrium occurs at the natural balance point of the system.*

Equilibrium Conditions:

$$\frac{dP}{dt} = 0, \quad \frac{dN}{dt} = 0 \tag{16}$$

Solution:

$$E_0 = (P^{\cdot} N) = \frac{\alpha_1 D + \alpha_2 S + \alpha_3 C}{\beta} \quad \frac{(\gamma_1 + \gamma_2 + \gamma_3)P}{* \ \delta} \tag{17}$$

1.7 Local Stability of the Low Fertility Equilibrium

Let $E_{LF} = (P^{\cdot} N^{)}$ denote the low fertility equilibrium. Linearizing the model around E_{LF} gives

$$\mathbf{y}^{\cdot} = J\mathbf{y}, \qquad \mathbf{y} = \begin{pmatrix} p \\ n \end{pmatrix} = \begin{pmatrix} P - P^* \\ N - N^* \end{pmatrix},$$

with Jacobian

$$J = \begin{pmatrix} -\beta & 0 \\ \gamma_1 + \gamma_2 + \gamma_3 & -\delta \end{pmatrix}, \quad \beta > 0, \ \delta > 0, \ \gamma_i \geq 0.$$

Characteristic Polynomial. The characteristic polynomial is

$$\chi_J(\lambda) = \det(J - \lambda I) = \det \begin{pmatrix} -\beta - \lambda & 0 \\ \gamma_1 + \gamma_2 + \gamma_3 & -\delta - \lambda \end{pmatrix} = (-\beta - \lambda)(-\delta - \lambda) = (\lambda + \beta)(\lambda + \delta).$$

Eigenvalues. Solving $\chi_J(\lambda) = 0$ yields

$$\lambda_1 = -\beta, \qquad \lambda_2 = -\delta.$$

Since $\beta, \delta > 0$, both eigenvalues are strictly negative.

Routh–Hurwitz Check (2D). For a 2×2 system, local asymptotic stability is equivalent to

$$\mathrm{tr}(J) < 0 \quad \text{and} \quad \det(J) > 0.$$

Here,

$$\mathrm{tr}(J) = -\beta - \delta < 0, \qquad \det(J) = (-\beta)(-\delta) - 0 \cdot (\gamma_1 + \gamma_2 + \gamma_3) = \beta\delta > 0,$$

so the equilibrium is locally asymptotically stable.

Gershgorin Discs (Sufficient Condition). The Gershgorin discs are

$$D_1 : \text{ center } -\beta, \text{ radius } 0 \Rightarrow \{\lambda_1 = -\beta\}, \qquad D_2 : \text{ center } -\delta, \text{ radius } \gamma_1 + \gamma_2 + \gamma_3.$$

If additionally $\delta > \gamma_1 + \gamma_2 + \gamma_3$, then D_2 lies entirely in the open left half-plane, giving a purely Gershgorin-based sufficient condition. (Regardless, the exact eigenvalues above already show stability for $\beta, \delta > 0$.)

Theorem 2.1 (Local Stability of E_{LF}). *With $\beta > 0$ and $\delta > 0$, the low fertility equilibrium E_{LF} is a hyperbolic sink (stable node) and hence locally asymptotically stable.*

Proof. The linearization matrix J has eigenvalues $\lambda_1 = -\beta$ and $\lambda_2 = -\delta$, both strictly negative. There- fore, the real parts of all eigenvalues are negative, and E_{LF} is locally asymptotically stable by the linearization principle. Equivalently, $\mathrm{tr}(J) < 0$ and $\det(J) > 0$ satisfy the Routh–Hurwitz conditions for 2×2 systems. □

Theorem 2.2 (Local Asymptotic Stability). *The low fertility equilibrium is locally asymptotically stable if and only if $\beta > 0$ and $\delta > 0$.*

Theorem 2.3. *If both eigenvalues of the Jacobian are negative, the equilibrium is locally asymptotically stable.*

Proof. Since both eigenvalues are negative (assuming positive parameters), the equilibrium is a stable node. □

1.8 Global Stability of Low Fertility Equilibrium

Theorem 2.4 (Global Asymptotic Stability). *If $\beta > 0$ and $\delta > 0$, then the equilibrium E_0 is globally asymptotically stable.*

Proof. Consider the Lyapunov function:

$$V(P, N) = \frac{1}{2}(P - P^*)^2 + \frac{1}{2}(N - N^*)^2 \tag{18}$$

$$\frac{dV}{dt} = (P - P^*)\frac{dP}{dt} + (N - N^*)\frac{dN}{dt} \tag{19}$$

$$= (P - P^*)(-\beta(P - P^*)) + (N - N^*)(-\delta(N - N^*) + \gamma(P - P^*)) \tag{20}$$

$$= -\beta(P - P^*)^2 - \delta(N - N^*)^2 + \gamma(N - N^*)(P - P^*) \tag{21}$$

For global stability, we need $\frac{dV}{dt} < 0$. This holds when:

$$\beta\delta > \frac{\gamma^2}{4} \tag{22}$$

where $\gamma = \gamma_1 + \gamma_2 + \gamma_3$. □

1.9 High Fertility (Endemic) Equilibrium

Enhanced Model with Feedback:

$$\frac{dP}{dt} = \alpha_1 D + \alpha_2 S + \alpha_3 C + \phi N - \beta P \tag{23}$$

Endemic Equilibrium:

$$P_e = \frac{\alpha_1 D + \alpha_2 S + \alpha_3 C + \phi N_e}{\beta} \tag{24}$$

$$N_e = \frac{\gamma P_e}{\delta} \tag{25}$$

Solving simultaneously:

$$N_e = \frac{\gamma(\alpha_1 D + \alpha_2 S + \alpha_3 C)}{\delta\beta - \gamma\phi} \tag{26}$$

1.10 Stability of the system

To analyze the stability of the system, we first need to compute the Jacobian matrix, find its eigenvalues, and then determine stability based on those eigenvalues;

Step 1: Compute the Jacobian Matrix

The system of differential equations is:

$$\begin{pmatrix} \frac{dP}{dt} \\ \frac{dN}{dt} \end{pmatrix} = \begin{pmatrix} \alpha_1 D + \alpha_2 S + \alpha_3 C - \beta P \\ \gamma_1 P_1 + \gamma_2 P_2 + \gamma_3 P_3 - \delta N \end{pmatrix}$$

The Jacobian matrix J is the matrix of first-order partial derivatives of the system with respect to the state variables P and N.

$$J = \begin{pmatrix} \frac{\partial}{\partial P}(\alpha_1 D + \alpha_2 S + \alpha_3 C - \beta P) & \frac{\partial}{\partial N}(\alpha_1 D + \alpha_2 S + \alpha_3 C - \beta P) \\ \frac{\partial}{\partial P}(\gamma_1 P_1 + \gamma_2 P_2 + \gamma_3 P_3 - \delta N) & \frac{\partial}{\partial N}(\gamma_1 P_1 + \gamma_2 P_2 + \gamma_3 P_3 - \delta N) \end{pmatrix}$$

Calculating the partial derivatives, we get:

$$J = \begin{pmatrix} -\beta & 0 \end{pmatrix}$$

$$\gamma_1 + \gamma_2 + \gamma_3 \quad -\delta$$

Step 2: Find Eigenvalues

The eigenvalues of the Jacobian matrix J are found by solving the characteristic equation:

$$\det(J - \lambda I) = 0$$

Substituting J and simplifying:

$$\begin{vmatrix} -\beta - \lambda & 0 \\ \gamma_1 + \gamma_2 + \gamma_3 & -\delta - \lambda \end{vmatrix} = 0$$

The determinant simplifies to:

$$(-\beta - \lambda)(-\delta - \lambda) = 0$$

Solving for λ, the eigenvalues are:

$$\lambda_1 = -\beta, \quad \lambda_2 = -\delta$$

Step 3: Stability Analysis

The stability of the system is determined by the signs of the real parts of the eigenvalues: If all eigenvalues have negative real parts, the equilibrium is stable.

If any eigenvalue has a positive real part, the system is unstable.

In this case, since both eigenvalues $\lambda_1 = -\beta$ and $\lambda_2 = -\delta$ are negative (assuming $\beta > 0$ and $\delta > 0$), the system is stable.

2 Analytical Solution of the Fertility Model

We consider the system of first-order differential equations:

$$\begin{pmatrix} \frac{dP}{dt} \\ \frac{dN}{dt} \end{pmatrix} = \begin{pmatrix} \alpha_1 D + \alpha_2 S + \alpha_3 C - \beta P \\ \gamma_1 P_1 + \gamma_2 P_2 + \gamma_3 P_3 - \end{pmatrix}$$

2.1 Solution for $P(t)$

Let

$$A = \alpha_1 D + \alpha_2 S + \alpha_3 C$$

so that the first equation becomes:

$$\frac{dP}{dt} = A - \beta P$$

This is a linear ODE. Using the integrating factor method:

$$\frac{dP}{dt} + \beta P = A$$

Multiplying both sides by the integrating factor $e^{\beta t}$:

$$\frac{d}{dt}(Pe^{\beta t}) = Ae^{\beta t}$$

Integrating both sides:

$$P(t)e^{\beta t} = \frac{A}{\beta}e^{\beta t} + C \Rightarrow P(t) = \frac{A}{\beta} + Ce^{-\beta t}$$

Applying the initial condition $P(0) = P_0$, we find:

$$C = P_0 - \frac{A}{\beta}$$

$$\boxed{P(t) = \frac{\alpha_1 D + \alpha_2 S + \alpha_3 C}{\beta} + \left(P_0 - \frac{\alpha_1 D + \alpha_2 S + \alpha_3 C}{\beta}\right) e^{-\beta t}}$$

2.2 Solution for $N(t)$

Let

$$B = \gamma_1 P_1 + \gamma_2 P_2 + \gamma_3 P_3$$

Then:

$$\frac{dN}{dt} = B - \delta N$$

This is also a linear ODE. Following the same procedure:

$$\frac{dN}{dt} + \delta N = B \Rightarrow \frac{d}{dt}(Ne^{\delta t}) = Be^{\delta t}$$

Integrating:

$$N(t)e^{\delta t} = \frac{B}{\delta}e^{\delta t} + C_1 \Rightarrow N(t) = \frac{B}{\delta} + C_1 e^{-\delta t}$$

Apply $N(0) = N_0$:

$$C_1 = N_0 - \frac{B}{\delta}$$

$$\boxed{N(t) = \frac{\gamma_1 P_1 + \gamma_2 P_2 + \gamma_3 P_3}{\delta} + \left(N_0 - \frac{\gamma_1 P_1 + \gamma_2 P_2 + \gamma_3 P_3}{\delta}\right) e^{-\delta t}}$$

2.3 Interpretation

Both $P(t)$ and $N(t)$ converge exponentially toward their respective equilibrium values:

$$P^* = \frac{\alpha_1 D + \alpha_2 S + \alpha_3 C}{\beta}, \quad N^* = \frac{\gamma_1 P_1 + \gamma_2 P_2 + \gamma_3 P_3}{\delta}$$

This reflects stabilizing behavior in proximate fertility determinants and cumulative fertility over time as shown in figure 2.

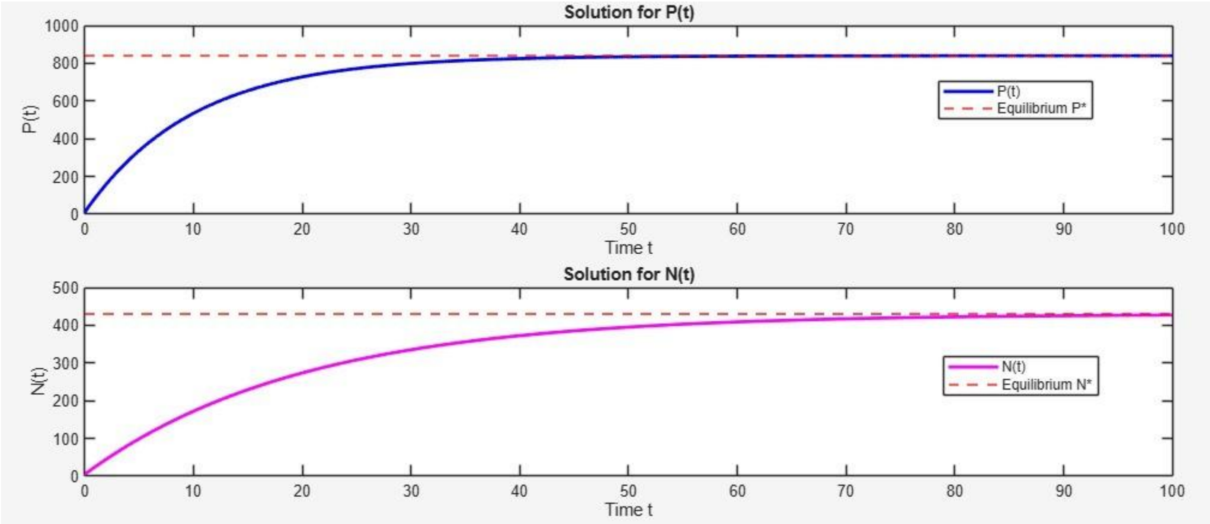

Figure 2: Solutions for P(t) and N(t)

2.3.1 Conclusion

i. The system will return to equilibrium after small perturbations, indicating stability in response to changes in the proximate determinants and background factors.

ii. This stability implies that fertility rates will eventually stabilize after any temporary fluctuations.

iii. The stable equilibrium confirms that fertility patterns respond to proximate determinants, but the system does not spiral out of control.

iv. Policy interventions can shift the equilibrium to a new, lower fertility rate.

v. Stability ensures long-term predictability, making the model useful for policy planning and demographic studies.

The framework supports evidence-based family planning policies, showing that fertility rates can be controlled through socio-economic and cultural interventions.

2.4 Sensitivity Analysis of R_0

From equation (15):

$$R_0 = \frac{(\gamma_1 + \gamma_2 + \gamma_3)(\alpha_1 D + \alpha_2 S + \alpha_3 C)}{\beta\delta} \tag{27}$$

2.4.1 Normalized sensitivity indices

:

$$\Upsilon^{\gamma_i}_{R_0} = \frac{\partial R_0}{\partial \gamma_i} \cdot \frac{\gamma_i}{R_0} = \frac{1}{3} \quad (\text{if } \gamma_1 = \gamma_2 = \gamma_3) \tag{28}$$

$$\Upsilon^{\alpha_i}_{R_0} = \frac{\partial R_0}{\partial \alpha_i} \cdot \frac{\alpha_i}{R_0} = \frac{\alpha_i X_i}{\alpha_1 D + \alpha_2 S + \alpha_3 C} \tag{29}$$

$$\Upsilon^{\beta}_{R_0} = \frac{\partial R_0}{\partial \beta} \cdot \frac{\beta}{R_0} = -1 \tag{30}$$

$$\Upsilon^{\delta}_{R_0} = \frac{\partial R_0}{\partial \delta} \cdot \frac{\delta}{R_0} = -1 \tag{31}$$

where $X_i \in \{D, S, C\}$.

2.4.2 Key Findings

i. **Most Sensitive Parameters**: β and δ (decay parameters) have sensitivity index of -1

ii. **Moderate Sensitivity**: Background factors (α_i) have variable sensitivity based on their relative contributions

iii. **Least Sensitive**: Individual fertility impact factors (γ_i) when equally weighted

2.4.3 Policy Implications

i. **High Impact Interventions**: Focus on strengthening regulatory mechanisms (increasing β, δ)

ii. **Targeted Interventions**: Address dominant background factors with highest $\alpha_i X_i$ products

iii. **Balanced Approach**: Simultaneously target multiple proximate determinants rather than focusing on one

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